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Magnetic circuits - electromagnet

Theme: *Electrical Machines*
→ ***Chapter:*** *Electromagnetic convertion*
→ ***Section:***

Type ressource: ☒ *Lecture* ☐ *Virtual laboratory / Exercice* ☐ *MCQ*

This lecture deals with the modelling of an electromagnet. This represents an opportunity for the introduction of Hopkinson's Law and the notion of magnetic circuit.

- prior knowledge:
- level: cycle 2
- auxiliary resources:
- estimated time :
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Education et culture

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1. The importance of the magnetic circuit notion

The electromagnetic torque T_{em} or the electromagnetic force F_{em} provided by an electromagnetic convertor, can be expressed as being the partial derivative, depending on the rotor or the relay armature's position and on the magnetic co-energy, the latter being expressed according to the currents which flow through the differrent windings of the electromagnetic convertor.

For a rotating convertor we can write :

$$T_{em} = \frac{\partial W_{cmag}}{\partial \theta_m}$$

For a lineary convertor:

$$F_{em} = \frac{\partial W_{cmag}}{\partial x_m}$$

The W_{cmag} magnetic co-energy can be calculated by integrating the fluxes expressions of the different windings depending on the flowing currents. For a *n* convertor we can write :

- in the case of a rotating convertor:

$$W_{cmag} = \int_{0,0,0,\ldots,0}^{i_1,i_2,\ldots,i_n} \sum_{k=1}^{n} \psi_k(i_1, i_2, \ldots, i_n, \theta_m) di_k,$$

- in the case of a lineary convertor:

$$W_{cmag} = \int_{0,0,0,\ldots,0}^{i_1,i_2,\ldots,i_n} \sum_{k=1}^{n} \psi_k(i_1, i_2, \ldots, i_n, x_m) di_k.$$

In the writing of the flux-currents relations it is, in some cases, required **the notion of magnetic circuit** (as well as **the notion of reluctance** which is associated to it), because this notion allows the direct deduction of $\psi_k(i_1,i_2,\ldots,i_n,\theta_m)$ or $\psi_k(i_1,i_2,\ldots,i_n,x_m)$ relations depending on the geometry of the studied device and on the permeability of the materials it is made of.

2. An example of magnetic circuit

To introduce the notion of magetic circuit, we will consider a reel with *N* turns reeled up round an annular core. (Figure 1).

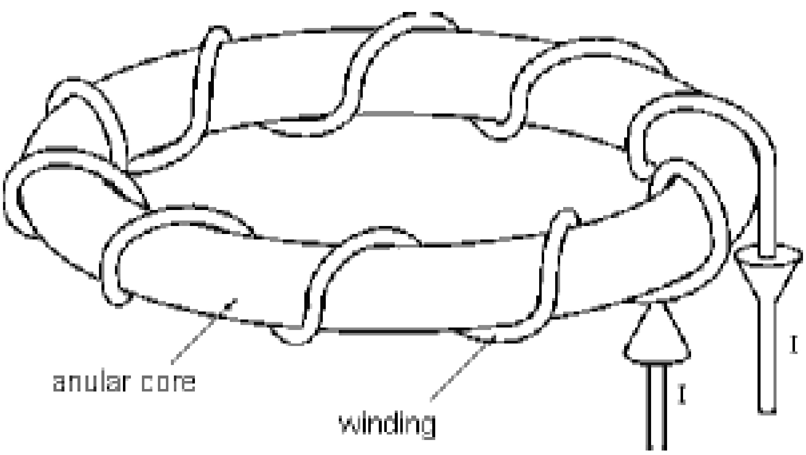

Figure 1

3. Defining the integration contour for the application of Ampère's theorem

The magnetic field created by the current which flows through the reel, can be calculated applying Ampère's theorem on circular contours situated in sections parallel to the reel, sections whose centres are on the symmetry axis of the reel. (Figure 2).

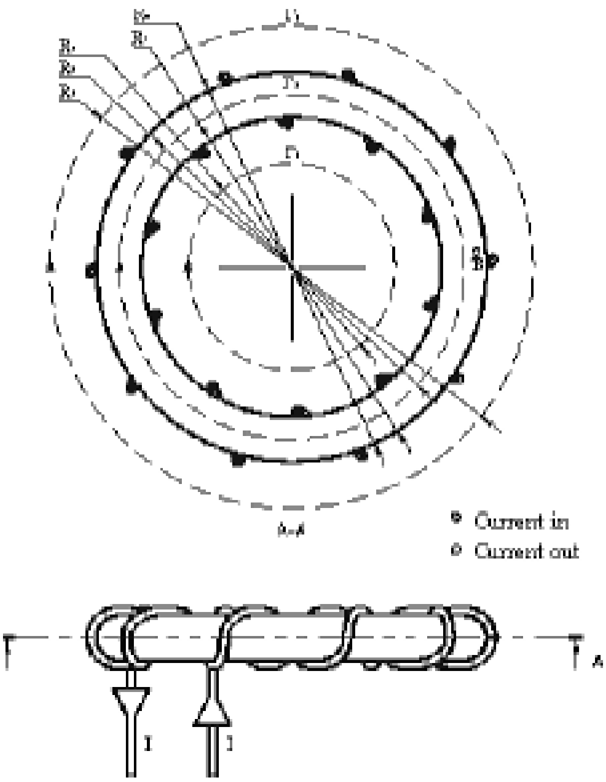

Figure 2

4. The application of Ampère's theorem - emphasizing the magnetic circuit

Due to the symmetry, on the chosen integration contours, the $\vec{B}$induction field produced by / current that flows through the reel is always tangential at contour and has constant amplitude. The result is:

- If the contour has the radius R_1 smaller than R_i (the internal radial of the annular core) - the Γ_1 contour from figure 2) [1]:

$$\oint_{\Gamma_1} \frac{\vec{B}}{\mu_0} . d\vec{l} = 2\pi \frac{R.B}{\mu_0} = 0; \qquad (1)$$

- If the contour has the radius R_2 bigger than R_i but smaller than R_e (the external radius of the annular core) the Γ_2 contour from figure 2):

$$\oint_{\Gamma_2} \frac{\vec{B}}{\mu} . d\vec{l} = 2\pi \frac{R.B}{\mu} = N.I; \qquad (2)$$

- finally, if the contour has the R_3 bigger than R_e - the Γ_3 contour from figure 2):

$$\oint_{\Gamma_3} \frac{\vec{B}}{\mu_0}.d\vec{l} = 2\pi \frac{R.B}{\mu_0} = N.I - N.I = 0; \qquad (3)$$

where μ_0 represents the magnetic permeability of the vacuum and of the air, and μ represents the magnetic permeability of the material from which the annular core is made.

It is ascertained that in any point outside the torus, the $\vec{B}$field is null. The entire flux induced by the current, circulates, therefore, within this volume, similar to an electrical circuit in which the power only flows through conductors. By analogy, we can define torus as being a **magnetic circuit**.

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[1] actually,for any contour situated in a plan which does intersect the reel.

5. Hopkinson's Law

If R_i şi R_e have similar values (meaning that the whirls are smaller than the medium radius $R_m = (R_i+R_e)/2$), we can consider, without risking major errors, that all the integration contours situated within the torus have the approximately same $R_m$lenght.

From here - the magnetic induction is constant in any point of a right section of the torus. As $\vec{B}$ induction is, in any point, perpendicularly to the right section (because it is tangential to the in tegration contour), the ψ flux through a right sectoin of the torus can be approximated:

$$\psi = \iint_S \vec{B}.d\vec{S} = B.S, \qquad (4)$$

where S is the right section of the torus.

Combining equations (2) and (4) the result will be

$$\psi = \frac{\mu.S}{\ell}.N.I, \qquad (5)$$

with $\ell = 2.\pi.R_m$

By noting and defining:

- $F = N.I$ magnetomotive force expressed in ampere-turns;
- $R = \frac{\ell}{\mu.S}$, reluctance of the magnetic circuit,

thus (5) could be write in this form:

$$\mathcal{F} = \mathcal{R}.\psi. \qquad (6)$$

This ecuation is also known as *The Hopkinson's Law*

6. Analogy between magnetic circuits/electrical circuits

Analogies between magnetic circuits and electrical circuits are easy to be made:

- The magnetic flux ψ which flows through a magnetic circuit, corresponds to the electrical current / which flows through an electrical circuit;
- the magnetomotive force *F*, corresponds to the electromotive force *U*;
- reluctance *R* of a magnetic conductor by ℓ lenght, *S* section and μ permeability, corresponds to resistance *R* of an electrical conductor by ℓ lenght, *S* section and σ conductivity; we can write *R* = $\frac{\ell}{\mu \cdot S}$ şi *R* = $\frac{\ell}{\sigma . S}$;
- finally Hopkison's Law *F* = *R*.ψ corresponds to Ohm's Law *U* = *R*.*I*.

We can also define the permeance of a magnetic circuit *P* = 1/*R,* which corresponds to *G* = 1/*R* conductance of an electrical circuit.

Magnetic circuit	*Electrical circuit*
Flux (ψ)	Current (*I*)
Magnetomotive force(*F*)	Electromotive force (*U*)
Reluctance (*R*)	Resistance (*R*)
Permeance (*P*)	Conductance (*G*)
Hopkison's Law (*F* = *R*.ψ)	Ohm's Law (*U* = *R*.*I*)

Analogy magnetic circuits / electrical circuits

7. Application for modelling an electromechanical converter

With the purpose of exemplification, we consider the application of the magnetic circuit notion for modelling the electromagnet in figure 3, for which we assume the same flux for both the ferromagnetic material and the electrical gap that separates them.

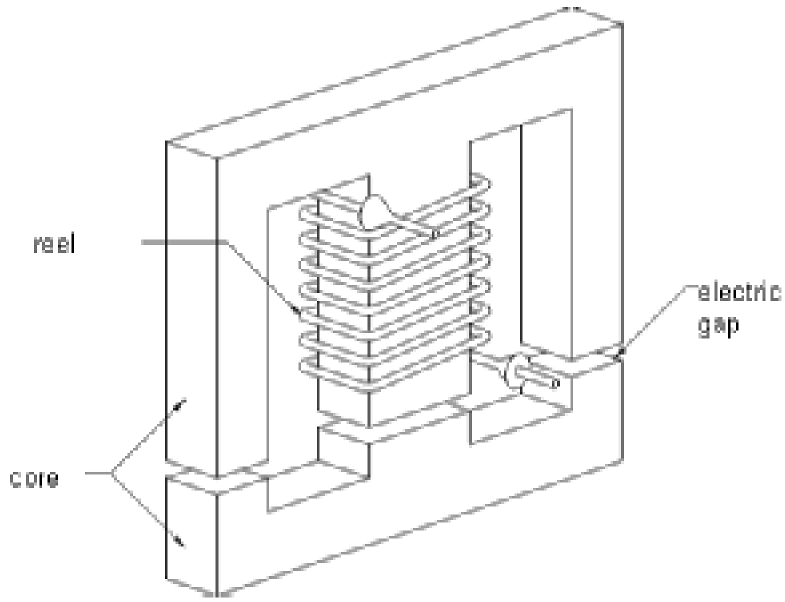

Figure 3

8. Defining the equivalent magnetic circuit

The analysis through the finite element method allows us to check the pertinency of the hypothesis, according to which the flux travels, mainly, through the ferromagnetic castings and the three electrical gap portions. Between two consecutive equiflux curves presented in figure 4, always travels the same quantity of magnetic flux.

The hypothesis we considered, is reduced, in fact, to the dereliction of the leakage flux (the one that does not travel the air-gap), a flux which gets smaller as the size of the air-gap that must be traveled gets smaller or as the relative permeability of the ferromagnetic material gets higher. [2].

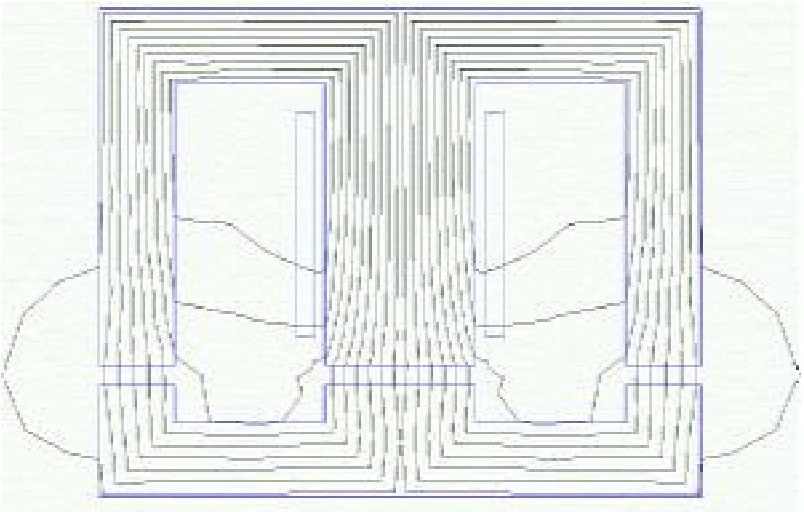

Figure 4

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[2] Which leads us to consider that ferromagnetic materials are unsaturated.

9. The reduction of the equivalent magnetic circuit

Considering the symmetry of the device, we can study only one half of the magnetic circuit (Figure 5). The fluxes that flow through the two lateral columns are equal, each being half of the flux that flows through the central column, and the flux which flows half of this central column's section. (Figure 5).

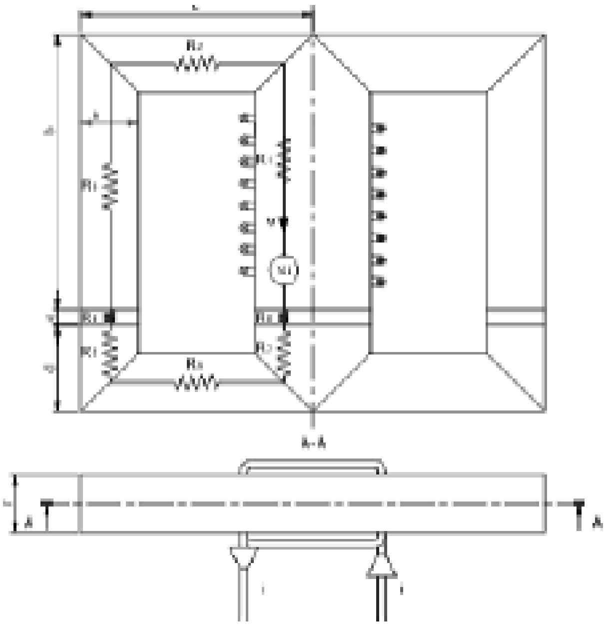

Figure 5

10. Flux-current relation

Knowing **the medium ℓ lenght** and **the perpendicular** *S* **section** of the different elements of the magnetic circuit as well as the μ permeability of the material they are made of, the nine reluctances of this circuit can be calculated by using the general relation

$$\mathcal{R} = \frac{\ell}{\mu.S}.$$

If μ_r is the relative permeability of the ferromagnetic material from which ($\mu = \mu_r \mu_0$) core is made of, permeability supposing to be constant whatever the value of the current (unsaturated circuit), we can calculate:

$$\mathcal{R}_1 = \mathcal{R}_3 = \frac{b - \frac{a}{2}}{\mu_r \cdot \mu_0 \cdot a \cdot f} \quad ;$$

$$\mathcal{R}_2 = \mathcal{R}_6 = \frac{c - a}{\mu_r \cdot \mu_0 \cdot a \cdot f} \quad ;$$

$$\mathcal{R}_4 = \mathcal{R}_8 = \frac{e}{\mu_0 \cdot a \cdot f} \quad ;$$

$$\mathcal{R}_5 = \mathcal{R}_7 = \frac{d - \frac{a}{2}}{\mu_r . \mu_0 . a . f} \quad .$$

The flux which flows through each of the two circuit branches (circuit equal to half of the total flux) can be, then, written as follows:

$$\frac{\psi}{2} = \frac{N.I}{\sum_{i=1}^{8} \mathcal{R}_i} = \frac{N \cdot I \cdot \mu_0 \cdot a \cdot f}{2e + \frac{2b + 2c + 2d - 4a}{\mu_r}} \quad . \qquad (7)$$

It is rather noteable that if the total lenght of the magnetic circuit is negligible in proportion to the product of μ_r and the total lenght of the air-gap portions, in order to obtain flux-current relation, it can be approximated, without making significant errors, that the total reluctance of the magnetic circuit is equal to that of the electrical gap portions [3]. For a relative μ_r permeability higher than 1000 and for values of the air-gap lower than 1mm, this approximation stays valid as long as the total lenght of the circuit remains lower than 2m.

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[3] **In fact, this kind of simplification is generally adopted in modelling the electromechanical converters , assuming that the permeability of the ferromagnetic materials they are made of, is infinite.**

11. The calculus of the magnetic co-energy and attraction force

The flux induced in the *N* whirls of the reel are expressed :

$$\Psi = N.\psi = L.I \, ,$$

in which according to (7),

$$L = \frac{N^2.\mu_0.a.f}{e + \dfrac{b+c+d-2a}{\mu_r}}.$$

The magnetic co-energy [4] is then

$$W_{cmag} = \int_0^i \Psi di_k = \frac{1}{2} L.I^2.$$

The attraction force between the two elements of the ferromagnetic core is finally expressed:

$$F_{em} = \frac{\partial W_{cmag}}{\partial e} = \frac{N^2.I^2.\mu_0.a.f}{2\left(e + \dfrac{b+c+d-2a}{\mu_r}\right)^2}.$$

It is in inverse ratio to *e*, having a maximum equal to:

$$F_{em,max} = \frac{N^2.I^2.\mu_r.\mu_0.a.f}{2(b+c+d-2a)^2}$$

for $e = 0$.

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[4] which is, however, equal to magnetic energy, as presented in §2.4.3 of the book because, assuming a constant μ_r, we face the case where the flux-current relations are linear.